AD-A102 593 NAVAL RESEARCH LAB WASHINGTON DC F/G 17/2.1
OPTIMUM SYNCHRONIZATION CODES TO FOLLOW AN ALTERNATING MARK/SPA--ETC(U)
JUL 81 J O COLEMAN
UNCLASSIFIED NRL-8494 NL

1 OF 1
AD A
102593

END
DATE
FILMED
9-81
DTIC

AD A102593

DTIC FILE COPY

12

NRL Report 8494

Optimum Synchronization Codes to Follow an Alternating Mark/Space Prefix

J. O. COLEMAN

Radar Analysis Branch
Radar Division

31 Jul[redacted]981

10

NAVAL RESEARCH LABORATORY
Washington, D.C.

Approved for public release; distribution unlimited.

251 950

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NRL Report 8494	2. GOVT ACCESSION NO. AD-A102593	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) OPTIMUM SYNCHRONIZATION CODES TO FOLLOW AN ALTERNATING MARK/SPACE PREFIX		5. TYPE OF REPORT & PERIOD COVERED Interim report on a continuing NRL problem
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) J. O. Coleman		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Research Laboratory Washington, DC 20375		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62712N; XF12151104 NRL Problem 53-0620-0
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Electronic Systems Command Washington, DC 20360		12. REPORT DATE July 31, 1981
		13. NUMBER OF PAGES 9
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Synchronization codes
Unique word
Barker codes
Burst communication systems

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Optimum synchronization codes of up to nine elements are given for a context in which the code is preceded in the received signal by a sequence alternating between +1 and −1. This context often appears in burst communication systems where the code is referred to as the "unique word."

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

CONTENTS

A

OPTIMUM SYNCHRONIZATION CODES TO FOLLOW AN ALTERNATING MARK/SPACE PREFIX

INTRODUCTION

Designers of data-transmission systems frequently need fixed sequences or codes a few bits long which can be inserted into binary data streams to facilitate synchronization in receiving equipment. Probably the most widely used codes for this purpose are the Barker codes [1-3], which have optimum autocorrelation functions if no signal is present either before or after the codes. Neuman and Hofman, using the same type of code quality measure, gave suboptimal codes [4] longer than the 13 bits of the longest Barker code. The Williard codes were designed for minimum probability of false sync if the code were embedded in random data [5].

We found that none of these codes were appropriate for our particular needs, which involved designing a modem for communication through a radar transmitter. We needed a code which would allow reliable synchronization with the code preceded with a prefix alternating between +1 and −1. This situation is commonly encountered in data-transmission modems which are required to operate in burst mode. The alternating sequence is provided to allow for the fastest possible acquisition of synchronization of bit or symbol timing at the modem receiver. The end of this synchronization preamble and the beginning of data proper are detected with a combination of a timeout from signal energy detection and a "match" indication from a binary matched filter looking for a predetermined end-of-preamble code a few bits long. It is assumed here that synchronization will be declared immediately upon recognition of the code in the data stream. The data following the code are therefore irrelevant.

This report gives the optimum codes of up to nine bits long for detection with a matched filter in such an environment. Massey mentioned in 1972 that the optimum synchronization rule for this situation remained an open problem [6]. To my knowledge it remains so, as this report does not examine the problem from the point of view of detection theory but simply assumes that the synchronization detector will look for a peak in the correlation of the N-bit code word with successively shifted N-bit segments of the received data stream.

THE OPTIMALITY CRITERION

The computation of the optimality criterion involves constructing a preamble consisting of B alternating bits followed by an N-bit code. This preamble is then put through a filter matched to the code only. The optimum codes are those that maximize the ratio of the largest peak in the filter output (when the matched filter contains the code) to the magnitude of the next largest preceding peak (the sidelobe magnitude peak) in the filter output. To express this mathematically, let $C(i)$, $0 < i < N + 1$, represent a code in which each $C(i)$ is equal to either +1 or −1. Construct from that code a sequence $S(i)$, $0 < i < 2N + B$, consisting of the preamble padded with leading zeros:

Manuscript submitted April 27, 1981.

$$S(i) = 0,\ 0 < i < N,$$

$$S(i) = (-1)^{i-N},\ N - 1 < i < N + B,$$

$$S(i) = C(i - N - B + 1),\ N + B - 1 < i < 2N + B.$$

The filter output function can then be defined by

$$F(k) = \sum_{i=1}^{N} C(i)\, S(k + i - 1),\ 0 < k < N + B + 1.$$

Because $F(N + B)$ will always be equal to N, the quantity to be minimized in choosing the code is the maximum of $|F(k)|$ over $0 < k < N + B$.

THE OPTIMAL CODES

The optimality criterion given above was applied to find the best codes of lengths two through nine. Table 1 gives the codes which were found to have sidelobe magnitude peaks of unity. For each code length, the table shows the preamble consisting of the alternating prefix followed by the code. The associated filter output is shown immediately below the preamble. The codes were found with a simple search technique performed by the program whose listing is shown in the Appendix. In all cases the number of alternating bits preceding the code was set to the smallest even number which was greater than or equal to the number of code bits. This makes the extrapolation of the given filter-output function to larger numbers of alternating bits simple.

Codes which have sidelobe peaks no greater in magnitude than unity were found only for lengths two, three, five, and seven. There are two such codes of length three, but only one for each

Table 1 — Codes with Sidelobe Magnitude Peaks of Unity

	Prefix	Code
Preamble	1 -1	1 1
Filter output	1 0	0 2
Preamble	1 -1 1 -1	1 -1 -1
Filter output	-1 0 1 -1	1 -1 3
Preamble	1 -1 1 -1	1 1 1
Filter output	1 0 1 -1	1 1 3
Preamble	1 -1 1 -1 1 -1	1 -1 -1 1 1
Filter output	1 0 -1 0 1 -1	1 -1 -1 -1 5
Preamble	1 -1 1 -1 1 -1 1 -1	1 -1 -1 1 1 1 1
Filter output	1 0 1 0 -1 0 1 -1	1 -1 -1 -1 1 1 7

of the other three lengths. For code lengths of four, six, eight, and nine, the sidelobe peak is of magnitude two. The codes for these four lengths are shown in Tables 2, 3, 4, and 5 respectively. There is more than one optimum code for each of these four lengths.

Presumably, the primary reason for using a code of this sort for synchronization is to make the synchronization process error tolerant. A single bit error can at worst raise the magnitude of the sidelobe peak by two and lower the actual correlation peak by two. Therefore, any code shown which has a correlation peak which exceeds the sidelobe magnitude peak by five or more can tolerate a single error. This condition is met by all of the codes shown of lengths seven and up.

Table 2 — Codes of Length Four

Prefix				Code			
1	-1	1	-1	-1	-1	-1	1
1	-2	1	-2	0	0	2	4
1	-1	1	-1	-1	-1	1	1
1	0	-1	0	-2	-2	2	4
1	-1	1	-1	-1	1	1	1
1	0	1	-2	0	-2	2	4
1	-1	1	-1	1	-1	-1	-1
-1	0	-1	2	-2	2	0	4
1	-1	1	-1	1	-1	-1	1
1	-2	1	0	0	0	-2	4
1	-1	1	-1	1	-1	1	1
1	0	-1	2	-2	2	-2	4
1	-1	1	-1	1	1	-1	-1
-1	0	1	0	0	-2	0	4
1	-1	1	-1	1	1	1	-1
-1	2	-1	2	-2	0	0	4
1	-1	1	-1	1	1	1	1
1	0	1	0	0	2	2	4

Table 3 — Codes of Length Six

Prefix						Code					
1	-1	1	-1	1	-1	-1	-1	-1	1	1	-1
-1	2	-1	0	-1	0	2	-2	0	-2	2	6
1	-1	1	-1	1	-1	-1	-1	1	-1	-1	1
1	-2	1	0	-1	0	-2	2	2	-2	0	6
1	-1	1	-1	1	-1	1	-1	-1	-1	-1	1
1	-2	1	-2	1	0	0	0	-2	2	0	6
1	-1	1	-1	1	-1	1	-1	-1	1	-1	-1
-1	0	1	-2	1	0	0	0	2	0	-2	6
1	-1	1	-1	1	-1	1	-1	-1	1	1	-1
-1	2	-1	0	-1	2	-2	2	0	-2	-2	6
1	-1	1	-1	1	-1	1	-1	-1	1	1	1
1	0	1	-2	1	0	0	0	-2	0	0	6
1	-1	1	-1	1	-1	1	-1	1	1	-1	-1
-1	0	1	0	-1	2	-2	2	-2	0	-2	6
1	-1	1	-1	1	-1	1	1	-1	-1	-1	-1
-1	0	-1	0	1	0	0	-2	0	0	2	6
1	-1	1	-1	1	-1	1	1	1	-1	-1	1
1	-2	1	0	1	0	0	2	-2	-2	0	6
1	-1	1	-1	1	-1	1	1	1	1	-1	-1
-1	0	1	0	1	0	0	-2	-2	0	2	6
1	-1	1	-1	1	-1	1	1	1	1	1	-1
-1	2	-1	2	-1	2	-2	0	0	2	2	6

Table 4 – Codes of Length Eight

Prefix								Code							
1	-1	1	-1	1	-1	1	-1	-1	-1	-1	1	1	-1	1	1
1	0	-1	2	-1	0	-1	0	-2	-2	0	-2	2	-2	2	8
1	-1	1	-1	1	-1	1	-1	-1	-1	1	-1	-1	-1	-1	1
1	-2	1	-2	1	0	-1	0	-2	2	2	2	0	0	2	8
1	-1	1	-1	1	-1	1	-1	-1	-1	1	-1	-1	1	1	1
1	0	1	-2	1	0	-1	0	-2	-2	-2	2	0	-2	2	8
1	-1	1	-1	1	-1	1	-1	1	-1	-1	-1	-1	1	-1	-1
-1	0	1	-2	1	-2	1	0	0	0	2	2	0	2	0	8
1	-1	1	-1	1	-1	1	-1	1	-1	-1	-1	-1	1	1	-1
-1	2	-1	0	-1	0	-1	2	-2	2	0	0	-2	0	0	8
1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	-1	-1	-1
-1	0	-1	0	1	-2	1	0	0	0	2	0	2	2	0	8
1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	-1	1	1
1	0	-1	0	1	-2	1	0	0	0	-2	0	2	-2	-2	8
1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	1	-1	-1	-1
-1	0	-1	2	-1	0	-1	2	-2	2	0	2	-2	-2	0	8
1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	1	1	1	-1
-1	2	-1	2	-1	0	-1	2	-2	2	0	-2	-2	0	0	8
1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	1	1	1	1
1	0	1	0	1	-2	1	0	0	0	-2	0	0	2	2	8
1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	-1	-1	-1
-1	0	-1	0	1	0	-1	2	-2	2	-2	0	-2	2	0	8
1	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1	1	1	1
1	0	1	0	-1	0	1	0	0	2	0	0	0	-2	2	8
1	-1	1	-1	1	-1	1	-1	1	1	-1	1	1	-1	-1	-1
-1	0	-1	2	-1	0	1	0	0	-2	0	-2	2	-2	0	8
1	-1	1	-1	1	-1	1	-1	1	1	1	-1	-1	1	-1	-1
-1	0	1	-2	1	0	1	0	0	-2	-2	2	0	-2	0	8
1	-1	1	-1	1	-1	1	-1	1	1	1	1	-1	-1	1	1
1	0	-1	0	1	0	1	0	0	2	2	0	-2	-2	2	8
1	-1	1	-1	1	-1	1	-1	1	1	1	1	-1	1	1	-1
-1	2	-1	0	1	0	1	0	0	-2	2	0	2	0	0	8
1	-1	1	-1	1	-1	1	-1	1	1	1	1	1	-1	-1	1
1	-2	1	0	1	0	1	0	0	2	-2	0	0	0	2	8

Table 5 — Codes of Length Nine

Prefix										Code								
1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	-1	-1	1	-1	-1	1
1	-2	1	0	-1	0	-1	0	1	-1	1	-1	-1	1	1	1	1	-1	9
1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	-1	1	1	1
1	0	1	-2	1	0	-1	0	1	-1	1	-1	-1	-1	-1	1	-1	-1	9

SUMMARY

Optimum synchronization codes of lengths two through nine have been given for a context in which the code is preceded in the received bit stream by a prefix of alternating +1 and -1. It was pointed out that the codes of lengths seven through nine can be detected in the received bit stream in the presence of a single bit error. Although our needs did not require optimum codes of lengths greater than nine, there is no reason to suppose such codes could not be easily found.

REFERENCES

1. R. H. Barker, "Group Synchronization of Binary Digital Systems," pp. 273-287 in *Communication Theory*, W. Jackson, editor, New York, Academic Press, 1953.
2. R. Turyn, "On Barker Codes of Even Length," Proc. IEEE 51, 1256 (Sept. 1963).
3. K. E. Toerper, "Biphase Barker-Coded Data Transmission," IEEE Trans. AES-4, 278-282 (Mar. 1968).
4. F. Neuman and L. Hofman, "New Pulse Sequences with Desirable Correlation Properties," pp. 277-282 in Nat. Telemetry Conf. 1971 Record.
5. M. W. Williard, "Optimum Code Patterns for PCM Synchronization," 1962 Nat. Telemetry *Conf.*, paper 5-5.
6. J. L. Massey, "Optimum Frame Synchronization," IEEE Trans. Comm. Tech. COM-20 (No. 2), 115-119 (Apr. 1972).

APPENDIX

APL Function Listings

```
    ∇ Z←ALT BEST B;⎕IO;M;C;N;S;K;F;P;I;ZINIT;PMIN
[1]   ⍝ ALT     NUMBER OF ALTERNATING BITS
[2]   ⍝ B       NUMBER OF CODE BITS
[3]   ⍝ Z       OPTIMUM SEQUENCES WITH FILTER OUTPUT FOR EACH
[4]    ⎕IO←1
[5]    M←⌊0.5+2*B ⍝                    NUMBER OF POSSIBLE CODES
[6]    C←¯1+2×(B⍴2)⊤¯1+⍳M ⍝            THE CODES, (⍴C) = B,M
[7]    N←ALT+B ⍝                       NUMBER OF BITS IN EACH SEQUENCE (BELOW)
[8]    S←((M,ALT)⍴ALT⍴ 1 ¯1),⍉C ⍝      SEQUENCES INCL PREAMBLE, (⍴S) = M,N
[9]    K←((⍳N)∘.+B-⍳N)∘.=⍳B ⍝          SIDELOBE CORRELATION KERNEL, (⍴K) = N,N,B
[10]   ZINIT←(0 2 ,N)⍴0 ⋄ Z←ZINIT ⋄ PMIN←B ⋄ I←1
[11] LOOP:F←S[I;]+.×K+.×C[;I] ⍝                 FILTER OUTPUT, (⍴F) = N
[12]   P←⌈/|¯1↓F ⍝                              PEAK SIDELOBE LEVEL, (⍴⍴P) = 0
[13]   →(~P<PMIN)/NEXT ⋄ PMIN←P ⋄ Z←ZINIT ⍝     NEW MINIMUM P
[14]   →(~P≤PMIN)/NEXT ⋄ Z←Z,[1] S[I;],[0.5] F ⍝ ANOTHER SEQUENCE AT OLD PMIN
[15]   I←I+1 ⋄ →(I≤M)/LOOP
    ∇

    ∇ Z←NEXT
[1]    Z←1+1↑1↓⎕LC ⍝ RETURNS NUMBER OF NEXT LINE IN CALLING FUNCTION
    ∇
```

DATE
FILMED

8